***Abstract***
Predictions about the future are contingent on future usage, but also on the quality of the models employed and the assessment of the current health state. These factors, amongst others, need to be considered to arrive at a prediction that is conducted through a rigorous method but where the confidence bounds are not prohibitively large.

# Prognostics Uncertainty Management

## with Applications to Government and Industry

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# Outline

- Fundamental Concepts
- Simple illustrative example
- Example on a Rover application
- Conclusion and recommendations

# Uncertainty in Prognostics and Health Monitoring

- Uncertainty is unavoidable in PHM
- Engineering system
  - Loading conditions and operating conditions are uncertain
  - Models used to study these systems are uncertain
- Health Management System
  - Sensors → errors due to bias and noise
  - Algorithms may not be accurate
- Leads to uncertainty in both diagnosis and prognosis
- Diagnosis → characterizing events that have happened
  - Uncertainty in fault detection, isolation, and estimation
  - Nevertheless, we can live without measures of uncertainty because we are only trying to find out what has already happened!
- Prognosis → characterizing events that are going to happen
  - How can we ever be certain about the future?



# Uncertainty in Prognostics

- Some of the existing literature claim to handle uncertainty in prognostics using particle filtering or other filtering approaches
  - Such a statement is incorrect
  - Filtering is only for state estimation, which may be considered a pre-cursor or first step to prognosis.
  - After state estimation, the goal in prognostics is to predict the future (no data)
  - Without data, the “correction step” of filtering will be absent, and therefore, it is not appropriate to call it “filtering” – we “filter” based on data!
  - In fact, the term “filtering for prognostics” is simply a misnomer
  - You simply take particles/samples and predict forward into future
  - Of course, if using Kalman filter, we have closed form expressions for future state distributions
- Another challenge is that of uncertainty interpretation – this is closely tied to the type of prognostics method and the context of our application
- A lot of existing literature seems to use “uncertainty management” for almost every task related to uncertainty and this is not quite correct.

# Types of Uncertainties

- Model uncertainties – Epistemic
  - Numerical errors
  - Unmodeled phenomenon
  - System model and Fault propagation model
- Input uncertainties – Aleatoric
  - Initial state (damage) estimate
  - Manufacturing variability
- Measurement uncertainties – Prejudicial
  - Sensor noise
  - Sensor coverage
  - Loss of information during preprocessing
  - Approximations and simplifications
- Operating environment uncertainties
  - Unforeseen future loads / environment
  - Variability in the usage history data

Systematic uncertainties due to things we could know in principle, but don't in practice.

Statistical uncertainties that may change every time the system is run.

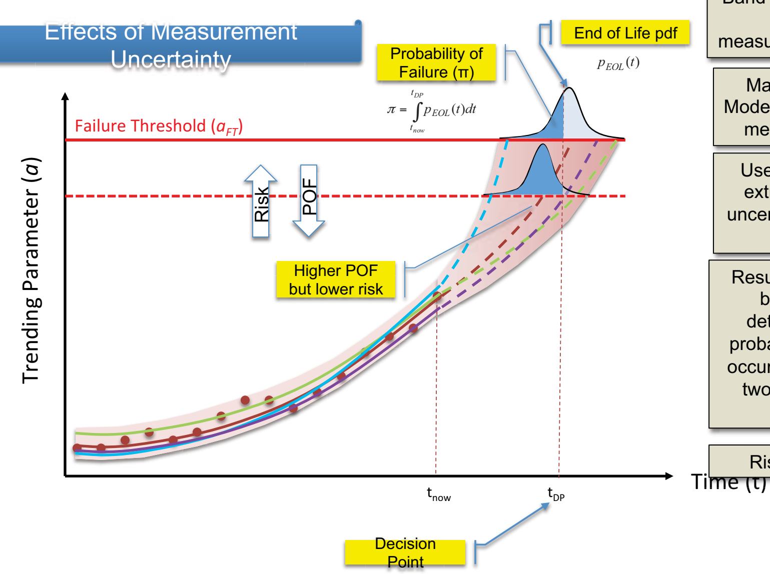
Unknown uncertainties due to the way data are collected or processed.

Can be a mix of any of the above.



# Trends and Thresholds Revisited

Effects of Measurement Uncertainty



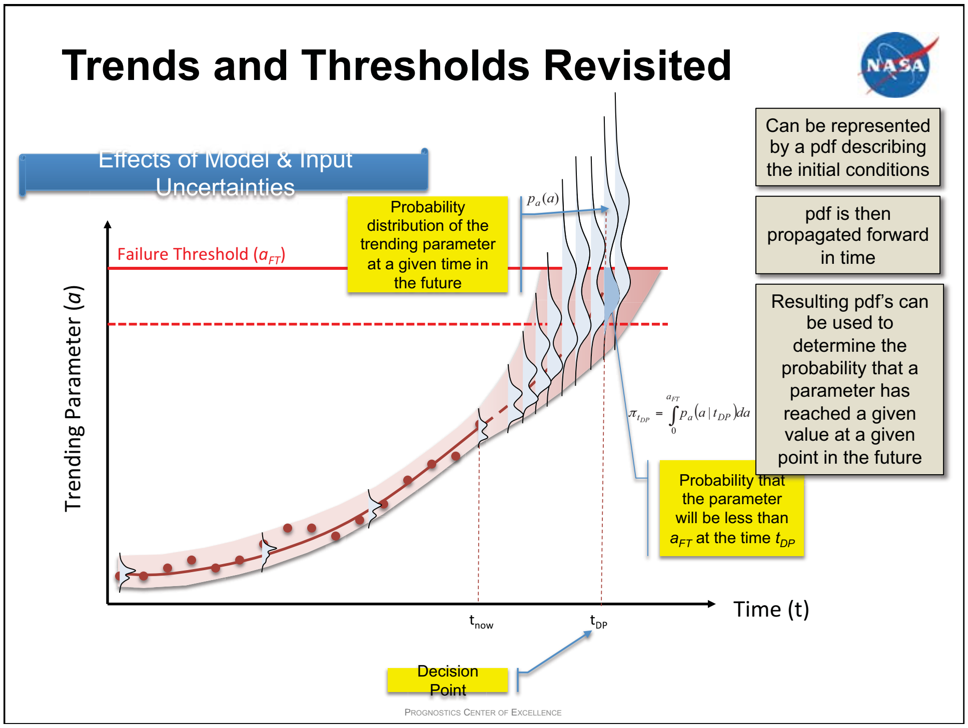

Band of uncertainty around measurement points

Many possible Models may "fit" the measurements

Use statistics to extrapolate the uncertainty into the future

Resulting pdf can be used to determine the probability of EOL occurring between two future time points

Risk vs POF


Trends and Thresholds Revisited
Effects of Model & Input Uncertainties
Probability distribution of the trending parameter at a given time in the future
$p_a(a)$
Failure Threshold ($a_{FT}$)
Trending Parameter ($a$)
$\pi_{t_{DP}} = \int_0^{a_{FT}} p_a(a \mid t_{DP})da$
Probability that the parameter will be less than $a_{FT}$ at the time $t_{DP}$
Can be represented by a pdf describing the initial conditions
pdf is then propagated forward in time
Resulting pdf's can be used to determine the probability that a parameter has reached a given value at a given point in the future
Time (t)
$t_{now}$
$t_{DP}$
Decision Point
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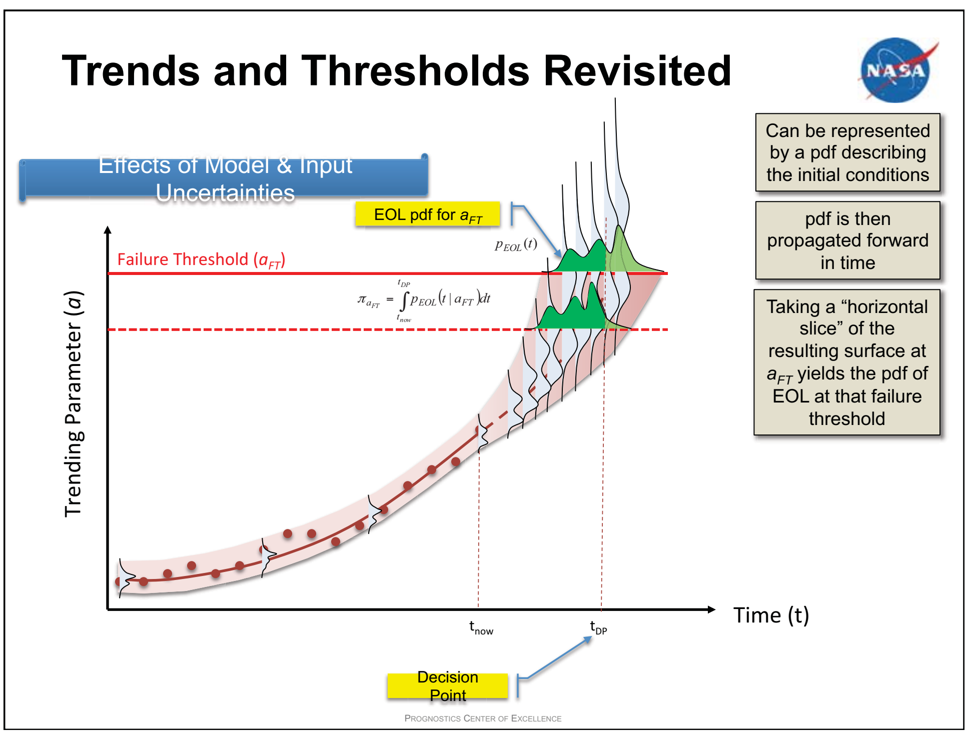
Trends and Thresholds Revisited
Effects of Model & Input Uncertainties
EOL pdf for $a_{FT}$
$p_{EOL}(t)$
Failure Threshold ($a_{FT}$)
$\pi_{a_{FT}} = \int_{t_{now}}^{t_{DP}} p_{EOL}(t \mid a_{FT})dt$
Trending Parameter ($a$)
Can be represented by a pdf describing the initial conditions
pdf is then propagated forward in time
Taking a "horizontal slice" of the resulting surface at $a_{FT}$ yields the pdf of EOL at that failure threshold
Time (t)
$t_{now}$
$t_{DP}$
Decision Point
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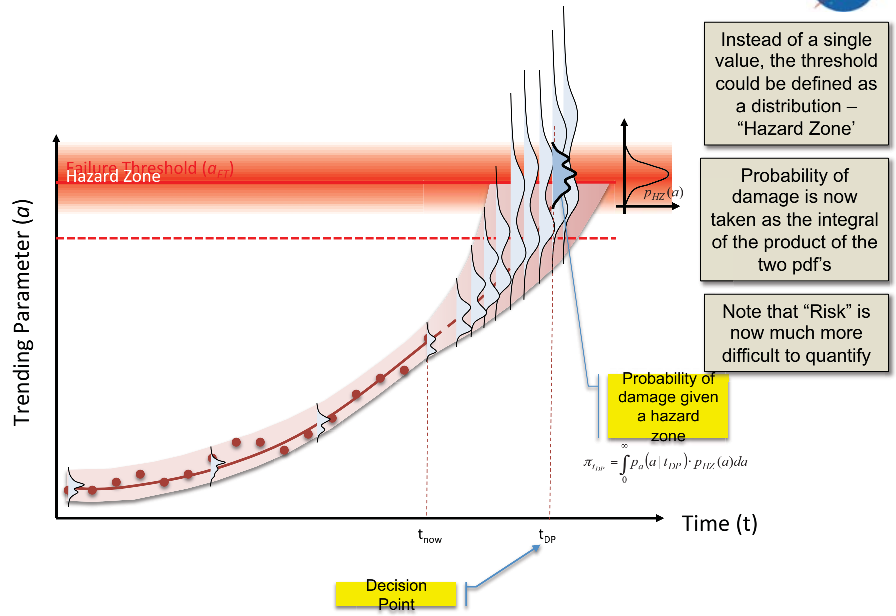
## Trends and Thresholds Revisited






## Uncertainty-Related Activities in Prognostics

- Uncertainty Representation and Interpretation
  - What method we choose to represent uncertainty?
  - Mostly, we have been using probability methods
  - Fuzzy logic, evidence theory, etc. are also available
  - Even for probabilistic methods, can we correctly interpret the uncertainty?
- Uncertainty Quantification
  - Identify each source of uncertainty that affect prognostics
  - State estimation uncertainty, future loading uncertainty, model uncertainty, etc.
  - Quantify each source of uncertainty individually
- Uncertainty Propagation
  - Quantify the combined effect on prognostics
  - Quantify uncertainty in future states and quantify uncertainty in RUL
- Uncertainty Management
  - How to reduce the uncertainty in RUL?
  - How to translate the information regarding uncertainty to the decision-maker?

# Uncertainty Interpretation

- Probabilistic methods are used in PHM
  - Bayesian filtering approaches are used to estimate uncertainty in the states → Particle, Kalman, Extended Kalman, Unscented Kalman …
  - A lot of initial studies assumed Gaussian (normal) variables for different quantities but now we are slowly moving over to non-Gaussian variables too, particularly in non-linear systems
  - We are familiar with probabilistic theorems, probability density functions, cumulative distributions, etc.
- But what do we mean by probability ?!
  - What do we mean by “The probability of a particular event is 0.4” ?



# Approaches to Prognostics

- Testing-based
  - Run multiple components/systems to failure
  - Collect several realizations of run-to-failure data
  - Use statistical tools for quantifying prognostics uncertainty
  - More suited to analyze reliability/prognostics of fleet of components/systems since the data is collected across multiple components of the fleet
  - Practically feasible only to small components since the cost of failing large systems is very high
- Condition-based
  - Focus is on one particular component or subsystem or system
  - Intra-fleet variability is not considered
  - At any time instant, the condition of the system is estimated (filtering) and RUL needs to be predicted
  - Applicable to ISHM, CBM

# Why is the RUL prediction uncertain?

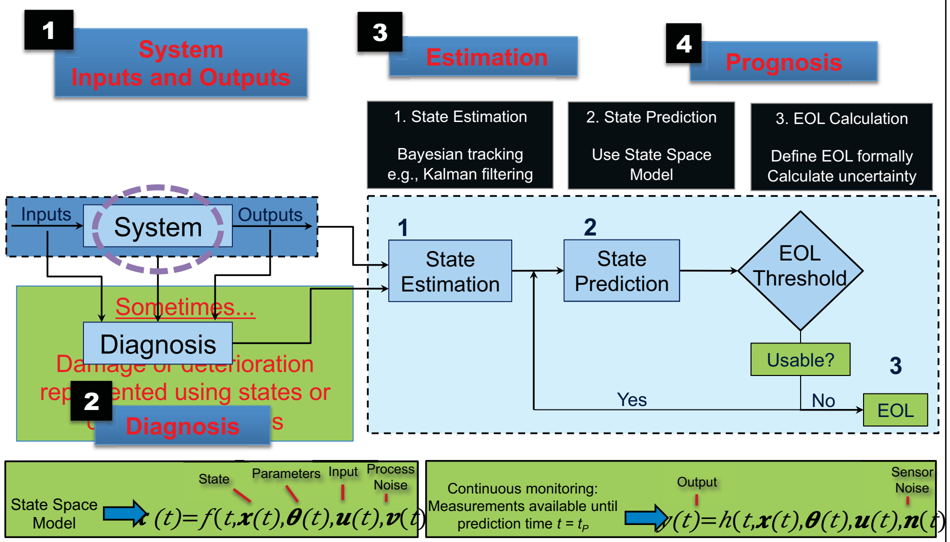
- Either because
  - There is true variability across multiple specimens/components
  - We do know certain deterministic quantities exactly and we assign subjective uncertainties to them
- Sources of Uncertainty
  - Present uncertainty: estimating present health state of the system
  - Future uncertainty: future operating and loading conditions
  - Prediction model uncertainty: model used to predict future state evolution
    - Model form, model parameters, process noise
  - Prediction method uncertainty: algorithm used to combine the different sources of uncertainty may not be perfect
- Uncertainty quantification → deals with quantifying the each of the above sources of uncertainty → a separate challenge by itself



# How to compute the uncertainty in RUL?

- For some reason, the uncertainty in RUL has been assumed to be Gaussian in many applications
  - In data-driven approaches, standard fitting techniques are used and there is some rational is assuming that the prediction is Gaussian
  - In model-based techniques, linear models are used along with Gaussian variables, and it is believed that this combination will lead to Gaussian RUL predictions
  - Well, both are wrong!!
- In data-driven approaches, the prediction uncertainty is mainly reflective of the points (i.e., data) used to "train" or "fit" the data-driven technique. More the points used, lower the uncertainty. This is not reflective of the underlying uncertainty – be it the result of "true variability" or "subjectivity"

# Model-Based Framework



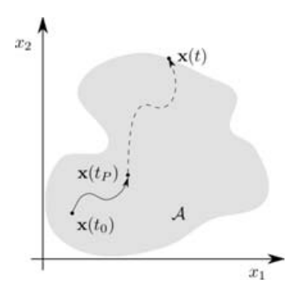



# Uncertainty in Prognostics & RUL

- Failure threshold $T_{EOL}$ is linked to boundary in multi-dimensional damage space
  - Beyond the boundary, the system does not conform to functional specifications (or operational risk is too great)
  - Within the boundary, the system is still functioning properly
- From time $t_0$ to prediction time $t_P$, system behaves within specs

$$EOL(t_P) = \inf\{t \in \mathbb{R} : t \geq t_P \wedge T_{EOL}(\boldsymbol{x}(t), \boldsymbol{\theta}(t)) = 1\}$$

$$RUL(t_P) = EOL(t_P) - t_P$$




$$\mathcal{A} = \{(\mathbf{x}(t), \boldsymbol{\theta}(t)) : T_{EOL}(\mathbf{x}(t), \boldsymbol{\theta}(t)) = 0\}$$

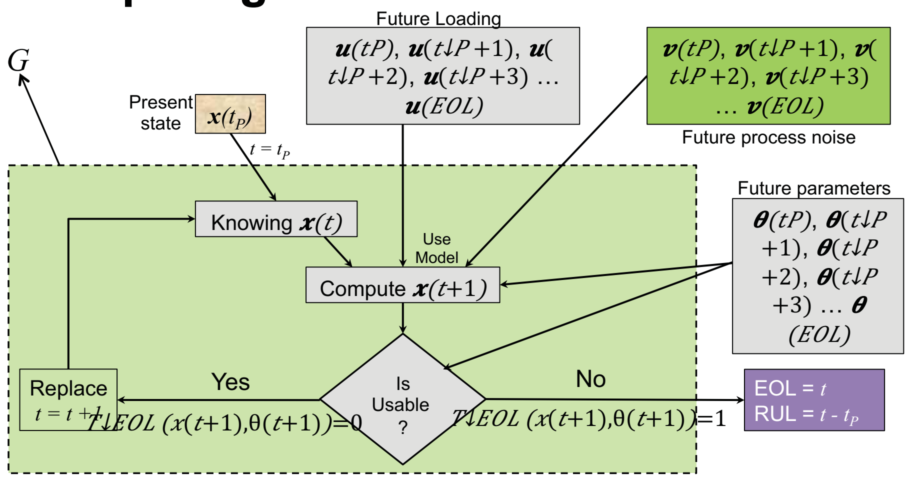
**EOL/RUL Prediction: Sources of Uncertainty**

- State at $t_p$ cannot be precisely known
- Future loading conditions and operating conditions not precisely known
- Even if state is known, future loading is known → we may not have the accurate "mechanism" to predict future state → model error

**Future behavior is uncertain**

- System takes some path *out of many possible paths* until EOL
- At any instant $t_P$, state evolution becomes a random process
- EOL and RUL are random variables

# Computing RUL




$RUL = G(x(tP),\ u(t↓P), u(t↓P+1), u(t↓P+2) … u(EOL),\ \theta(t↓P), \theta(t↓P+1), \theta(t↓P+2) … \theta(EOL),\ v(t↓P), v(t↓P+1), v(t↓P+2) … v(EOL))$



# Compute the uncertainty in RUL

$RUL = G(x(tP),\ u(t↓P), u(t↓P+1), u(t↓P+2) … u(t↓P+RUL),\ \theta(t↓P), \theta(t↓P+1), \theta(t↓P+2) … \theta(t↓P+RUL),\ v(t↓P), v(t↓P+1), v(t↓P+2) … v(t↓P+RUL))$

$R = G(\mathbf{X}) = G(X↓1, X↓2, X↓3 … X↓n)$

$X↓i \sim f↓X↓i\ (x↓i)$

- Computing RUL uncertainty in basically propagating the uncertainty in $\mathbf{X}$ through "$G$"
  - Closed form expressions not available even for certain simply scenarios
- Given the full probability distribution of each $X_i$ → Compute distribution of RUL
- Several methods available for such uncertainty propagation
  - Sampling techniques
  - Analytical techniques
  - Mixed approaches
- Points to Remember
  - Methodology needs to be feasible for online implementation
  - Accurately capture the distribution shape (Even multi-modal RUL distributions)
  - Deterministic methods are preferred → leads to the next presentation
  - Capturing central behavior is easy → also capture probability tails
  - Easily calculate some sort of "bounds" on RUL without much computational effort

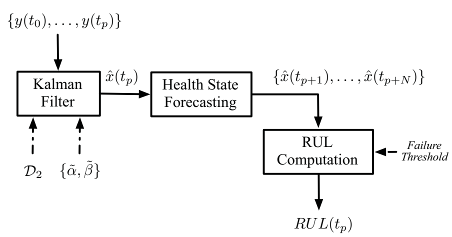
# Simple illustrative example



# Model-based prognostics (1/2)

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), u(t)) + w(t)$$
$$y(t) = h(\mathbf{x}(t)), u(t)) + v(k)$$

$$R(t_p) = t_{EOL} - t_p$$




- State vector includes dynamics of the degradation process
- It might include nominal operation dynamics
- EOL defined at time in which performance variable cross failure threshold
- Failure threshold could be crisp or also a random variable

## RUL as a random process

- Random process: Family of random variables indexed in time
  - RUL estimates through time can be regarded as a random process
  - *R(t):* RUL estimation at time t, is a random variable
  - The probability density function (PDF) is unknown
  - Prognostics algorithm attempts to estimate parameters of *R(t)* or to approximate the density altogether



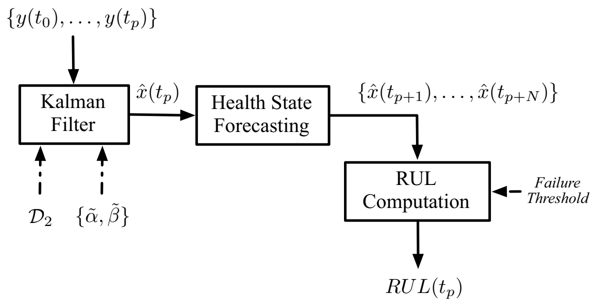
## Implications on health state estimation (1/2)

- Measurement and model noise variances become important
  - They need to be representative of the real process as for the uncertainty represented on the state estimate to be accurate
- Variance of sensor error can be computed if sensor is available for experimentation
- Variance of model error could be computed from run-to-failure data and degradation model
  - This is possible for simple cases but a challenge for large systems

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), u(t)) + w(t)$$
$$y(t) = h(\mathbf{x}(t)), u(t)) + v(k)$$

## Implications on health state estimation (2/2)

- Uncertainty in the health state tracking should be properly represented
- Uncertainty needs to be propagated through the forecasting process and to the RUL computation process





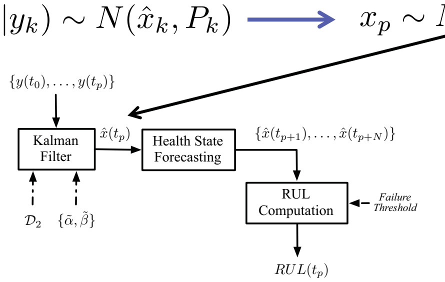

## Kalman filter background

- The optimal state estimate is the conditional expectation

$$x^*(t) = E[x(t)|y(t_o), \cdots, y(t)]$$

- Assuming a scalar discrete-time case. The distribution of the state is as follows

$$p(x_k|y_k) \sim N(\hat{x}_k, P_k) \longrightarrow x_p \sim N(\hat{x}_p, P_k)$$



# Uncertainty propagation (1/2)

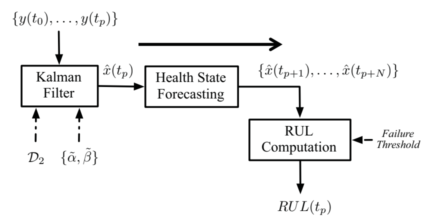
- The forecast are computed using the state equation only (one step ahead)

$$\tilde{x}_p(1) = Ax_p + Bu_p + w_p$$

- $l^{th}$ step ahead forecasting:

$$\tilde{x}_p(l) \sim N(\mu_l, {\sigma_l}^2)$$

$$\mu_l = A^l \hat{x}_p + Bu_p + \sum_{i=0}^{l-1} A^i \quad {\sigma_l}^2 = (A^2)^l P_k + \sum_{i=1}^{l-1} (A^2)^i {\sigma_w}^2 + {\sigma_w}^2$$





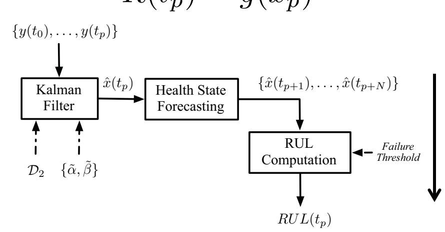

# Uncertainty propagation (2/2)

- The last step is to propagate to

$$R(t_p) = t_{EOL} - t_p$$

- This is not easy to do analytically as the previous steps. Consider as a function of random variable, one could use a computational method

$$R(t_p) = g(x_p)$$



## Discussion(1/2)

- From analytical results, variance will be higher after forecasting process as expected
  - The more forecasting steps, the larger the variability
  - This is consistent with the notion that the estimation should become more accurate as the component gets closer to EOL

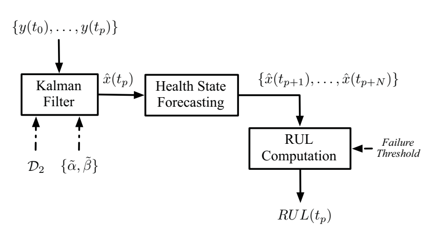
$$\sigma_l^{\,2} = (A^2)^l P_k + \sum_{i=1}^{l-1} (A^2)^i \sigma_w^{\,2} + \sigma_w^{\,2}$$

- There is no evidence to support RUL being Normal so this should be avoided



## Discussion (2/2)

- Non-linear random variable transformation involved in final step
  - Computational method could solve this
  - Exploring if it can be solved analytically
  - It is evident that it is incorrect to assume that RUL variable is the same as the error covariance in the state estimation or the variance after the forecast
- This assessment was conducted over a scalar case.
  - Additional considerations should be taken on the vector case



# Example on a Rover application



# Case Study
# Power System of a Planetary Rover

- Battery prognostics for a planetary rover, simulation-based results
- Event E is end-of-discharge, defined as the time when the battery voltage reaches 2.5 V
- Rover has set of waypoints that it is traversing

- Two different scenario types:
  - Rover commanded to go the same forward speed between waypoints (analogous to constant loading)
  - Rover commanded to go different speeds between waypoints (analogous to variable loading)

# Uncertainty Representation

- Knowing present state at prediction-time $t_P$
  - Provided by the state estimation algorithm
  - Particle filter → particles + weights
  - Kalman filter → statistics of Gaussian (normal) distribution
  - Use such information to calculate the PDF of the state variable
- The other three quantities are time-trajectories
  - Future Loading
  - Future parameter values
  - Future process noise values
- At each time instant, they follow a probability distribution
  - Quantifying this, by itself, is a separate topic of research
  - Assume that this is available for this research work
  - Focus on quantifying their combined effect on prediction uncertainty
- Concept: stochastic process



# Representation of Stochastic Processes

- Stochastic process
  - At each time instant, the quantity follows a probability distribution
  - A stochastic process can be expressed as a combination of deterministic time-dependent function and random time-independent variables
  - For example, the concept of Karhunen-Loeve expansion
  - Following a similar, simpler approach in this work
- Stochastic process → "deterministic" time-dependent function + time-invariant random variables
  - Since these random variables can reproduce the stochastic process, they are referred to as "surrogate variables"
  - For e.g., choose surrogate variables
  - Then, the trajectory can be defined
  - Easier to define statistics for surrogate variables
  - For example, if future input is constant, it is defined variable which defines what the input value is for all future time steps

$$\boldsymbol{\lambda}_a = [\lambda_a^1 \lambda_a^2 \ldots]$$

$$\mathbf{A}_k(k) = \lambda_a^1 + \lambda_a^2 \sin k$$

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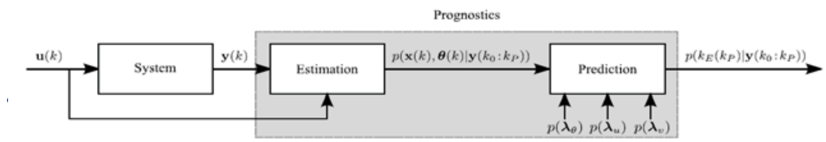
# Prognostics Architecture

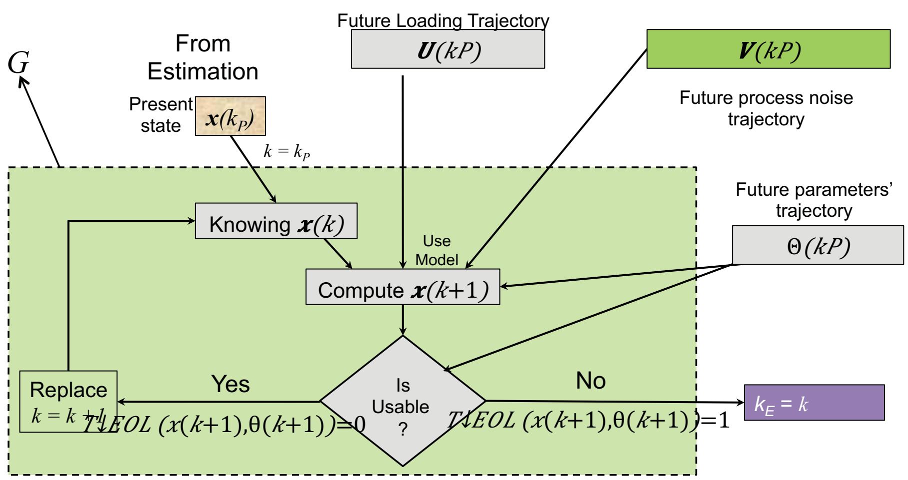
- Estimator → estimates of states
- This is an input to the prediction algorithm, along with the statistics of the surrogate variables, that correspond to
  - Future loading
  - Future parameter values
  - Future process noise values




  - Given the probability distributions of the present state values (indicate present health) and of the surrogate variables



# Prediction Procedure




$RUL = P(x(tP),\ \boldsymbol{u}(t\downarrow P),\boldsymbol{u}(t\downarrow P+1), \boldsymbol{u}(t\downarrow P+2) \ldots \boldsymbol{u}(EOL),\ \boldsymbol{\theta}(t\downarrow P),\boldsymbol{\theta}(t\downarrow P+1),\boldsymbol{\theta}(t\downarrow P+2) \ldots \boldsymbol{\theta}(EOL),\ \boldsymbol{v}(t\downarrow P),\boldsymbol{v}(t\downarrow P+1),\boldsymbol{v}(t\downarrow P+2) \ldots \boldsymbol{v}(EOL))$

## Compute prediction uncertainty

$$RUL = P\left(x(t_P),\ \boldsymbol{u}(t_P), \boldsymbol{u}(t_P+1), \boldsymbol{u}(t_P+2) \ldots \boldsymbol{u}(t_P+RUL),\ \boldsymbol{\theta}(t_P), \boldsymbol{\theta}(t_P+1), \boldsymbol{\theta}(t_P+2) \ldots \boldsymbol{\theta}(t_P+RUL),\ \boldsymbol{v}(t_P), \boldsymbol{v}(t_P+1), \boldsymbol{v}(t_P+2) \ldots \boldsymbol{v}(t_P+RUL)\right)$$

$$\rightarrow R = P(\boldsymbol{X}) = G(X_1, X_2, X_3 \ldots X_n)$$

$$X_i \sim f_{X_i}(x_i)$$

- Computing RUL uncertainty in basically propagating the uncertainty in $\boldsymbol{X}$ through "$P$"
  - Closed form expressions not available even for simple scenarios
  - The combination of state space models and the threshold function automatically renders "$P$" non-linear
- Given the full probability distribution of each $X_i$ → Compute distribution of RUL
- Choice of methods available for such uncertainty propagation driven by goals
  - Develop an efficient, integrated health monitoring system
  - Verification, Validation, and Certification of Systems
  - General consensus (based on standards) → "deterministic algorithms" to be used in ISHM
  - Whether is it possible construct "some" bounds on RUL!
- Standards for Aviation
  - Do not discuss much about uncertainty while prognostics uncertainty is unavoidable
  - We want the standards to evolve to consider non-deterministic approaches but will take time
  - Uncertainty quantification methods to be V&V-ed and certified → "deterministic" approaches
- Approaches presently pursued
  - Monte Carlo Sampling (non-deterministic)
  - Unscented transform sampling (deterministic)
  - Inverse First-order Reliability Method (or) Inverse-FORM (deterministic)



## Each method focuses on one aspect of the prediction

- Monte Carlo sampling
  - Attempt to directly generate samples from the distribution of $k_E$ we want to obtain → histogram, kernel density, etc. to get PDF
- Unscented transform sampling
  - Commonly used in Kalman filtering for state estimation
  - Sampling based on predetermined probability levels → sigma points
  - Estimate mean and variance with reasonable accuracy and hence suitable for state estimation in Kalman filtering
  - Also been used for prediction, but cannot get tail probabilities accurately
- Inverse FORM
  - Originally developed by structural engineers to calculate the probability of failure of structural systems
  - Focuses on estimating the inverse CDF of RUL
  - Calculate "$k_E$" given "$\eta$" , i.e. for a given probability level
  - Estimate the entire CDF of "$k_E$" by repeating for multiple probability levels

# Case Study: Battery Model

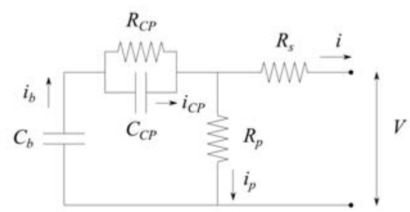
- Lithium-ion battery model with electrical circuit equivalent
  - $C_b$ holds battery charge

$$SOC = 1 - \frac{q_{max} - q_b}{C_{max}}$$

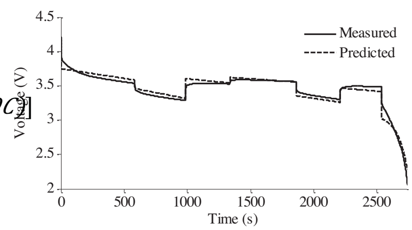
  - $R_{CP}/C_{CP}$ pair for concentration polarization (major nonlinear process producing steep discharge curve near end of discharge (EOD)

$$R_{CP} = R_{CP0} + R_{CP1} \exp[R_{CP2}(1 - SOC)]$$

- Circuit elements represent equilibrium voltage and voltage drops due to battery internal resistances and electrochemical processes




**Battery Equivalent Circuit**




**Battery Model Validation**



# Future Input Characterization

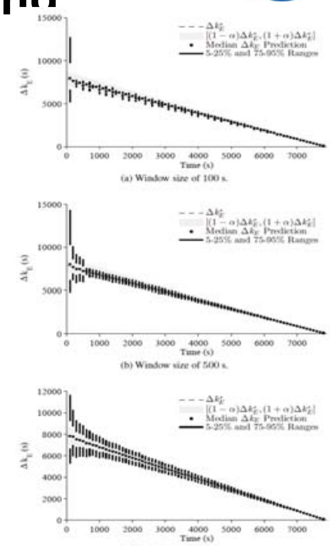
- Consider only uncertainty in state and future inputs
- While the state uncertainty is provided by the estimator (Unscented Kalman filtering is used in this work), future input trajectories need to be defined
- Surrogate variables corresponding to the future input trajectory need to be defined
- Future input to battery depends on how rover will be used – rover turns to next waypoint while maintaining desired forward speed. Turns take more power than going straight, and increase distance traveled between two waypoints over straight-line distance
- System-level approach accounts for all these details by simulating the entire rover instead of just the battery – most accurate but requires most computation
- Limit to the rover battery in this work without full system simulation

# Future Input Characterization (cont.)

- Approach 1
  - Assume that distribution of future inputs is same as past inputs
  - Maintain a window of past inputs in which to compute these statistics
  - Vary the window length
- Approach 2
  - Can incorporate some system-level knowledge while still having predictions local only to the battery
  - Compute power draw between waypoints for the known speed that the rover will go between them
  - Estimate future input trajectory based on current rover location and remaining waypoints
  - Improved future loading characterization



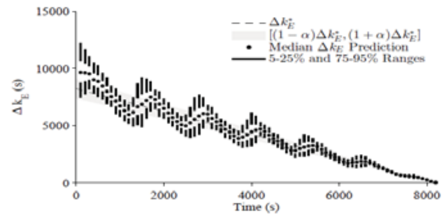
# Quantifying Prediction Uncertainty Results: Constant Loading



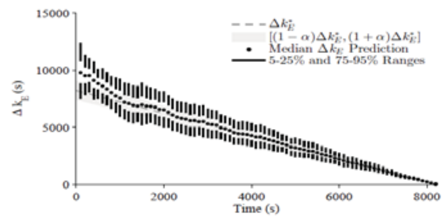

- Plots show prediction results using Monte Carlo sampling for the constant loading scenario
- Future loading looks like past loading
- As window size increases, uncertainty is better represented and bounds get larger
- If window is too small, then fluctuates between power draw for going straight and for turning (because window only captures one of these movements)
- UT and Inverse FORM results are virtually identical

# Quantifying Prediction Uncertainty Results: Variable Loading

- For variable loading scenario
- Using the window approach (assuming future inputs look like past inputs)
- Produces unstable predictions
- Because the assumption is violated → future inputs do not look like past inputs




(a) Window size of 500 s.




(b) Unlimited window size.

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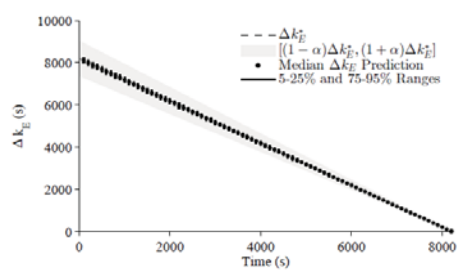
# Quantifying Prediction Uncertainty Results: Variable Loading

- Using improved method of future input characterization
- For each waypoint segment, surrogate variables are power draw and distance traveled
- Obtain much more accurate predictions that are also stable (without oscillation)

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# Conclusion and Recommendation



## Implications on decision-making (1/2)

- Properly capturing uncertainty is very important in prognostics
  - Decisions based on prognostics information might involve high risk
    - Contingency management
- Decision-making process in the application also guides how RUL is modeled and how uncertainty management is done
  - This is typically not done now but it is important to be considered
    - Control reconfiguration decision differ from inventory management decisions

## Implications on decision-making (2/2)

- Decision-making capabilities differ in terms of handling uncertainty on RUL information
  - Optimization on the decision process
    - Ability to deal with RUL as a random variable
    - Assume RUL as Normal or other parametrized distribution
    - Ability to deal with non-parametric estimate information
    - Ability to deal with a priori information about the quality of the estimation process



## Recommendations

- Caution on uncertainty reduction by algorithm tuning
  - Empirical RUL PDF variability vs. Point estimates variability
- Think about how the RUL estimation will be validated
  - Will help understand if uncertainty is properly taken into account
- Think about the post-prognostics decision making process
  - Can the system reason about empirical PDF or about non-normal random variables?
- Uncertainty management term is too ambiguous in the literature. Do not consider this a solved problem, it is easy to be misled, be inquisitive in establishing causality and validity of RUL uncertainty